A Proposed Method for Detection of Thermospheric Density Variations via Spacecraft Accelerations Observed Using the CASSIOPE GAP^2 Instrument

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Overview

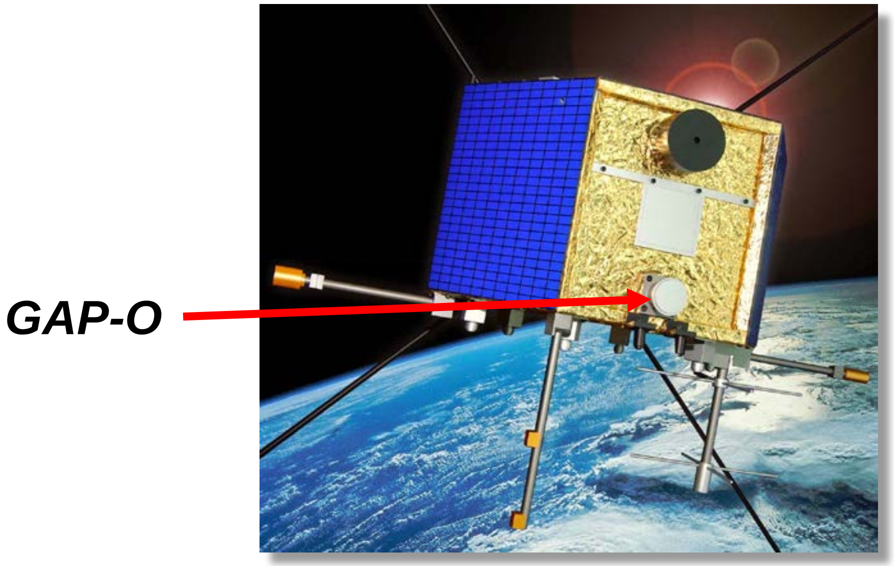
- CASSIOPE
- Error Analysis of Estimation Techniques
- GNSS-Based Velocity Estimation
    - Position change
    - Receiver-generated Doppler
    - Carrier-phase-derived Doppler (TDCP)
- Proposed Position/Velocity/Acceleration Algorithm
- Thermospheric Density Variation

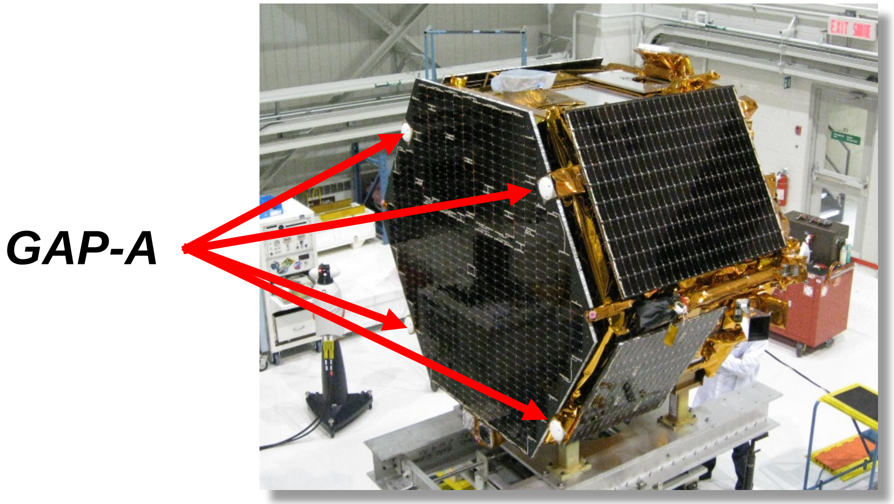
CASSIOPE

- CASSIOPE: CAScade, Smallsat, and IOnospheric Polar Explorer
  - Cascade: High data rate, high-capacity store-and-forward technology payload
  - e-POP: Enhanced Polar Outflow Probe
    - Coherent EM Radio Tomography (CER)
    - Fast Auroral Imager (FAI)
    - Imaging and Rapid-Scanning Ion Mass Spectrometer (IRM)
    - Fluxgate Magnetometer (MGF)
    - Neutral Mass Spectrometer (NMS)
    - Radio Receiver Instrument (RRI)
    - Suprathermal Electron Imager (SEI)
    - **GPS Attitude, Positioning, and Profiling experiment (GAP^2)**

GAP^2

- 5 dual-frequency GPS receivers/antennas
  - GAP-O
    - Anti-ram (anti-velocity) mount
    - High-rate (20 Hz) occultation data (standard)
      - Higher rates (50 Hz, 100 Hz) possible
  - **GAP-A**
    - Zenith-facing mount
    - 3 receivers in simultaneous operation
      - 1 spare (20 Hz), also useable for GAP-O
    - 1 Hz data rate (standard)
      - Higher rate (20 Hz) possible





Error Analysis of Estimation Techniques

- Propagation of Variances

  - $\Sigma_{xx} = \left(J^T \Sigma_{yy} J\right)^{-1}$

$$where \quad J = Jacobian\ matrix\ of\ linearized\ observation\ equations$$

$$\Sigma_{yy} = Observable\ variance/covariance\ matrix$$

$$\Sigma_{xx} = Parameter\ variance/covariance\ matrix$$

  - Provides estimates of achievable parameter accuracy, given:
    - Appropriate mathematical models
    - Realistic a priori estimates of observable quality

Velocity Estimation

- Position Change (delta position)

  - $V_{avg} = \frac{R_r(t_{k+\Delta t}) - R_r(t_k)}{t_{k+\Delta t} - t_k}$

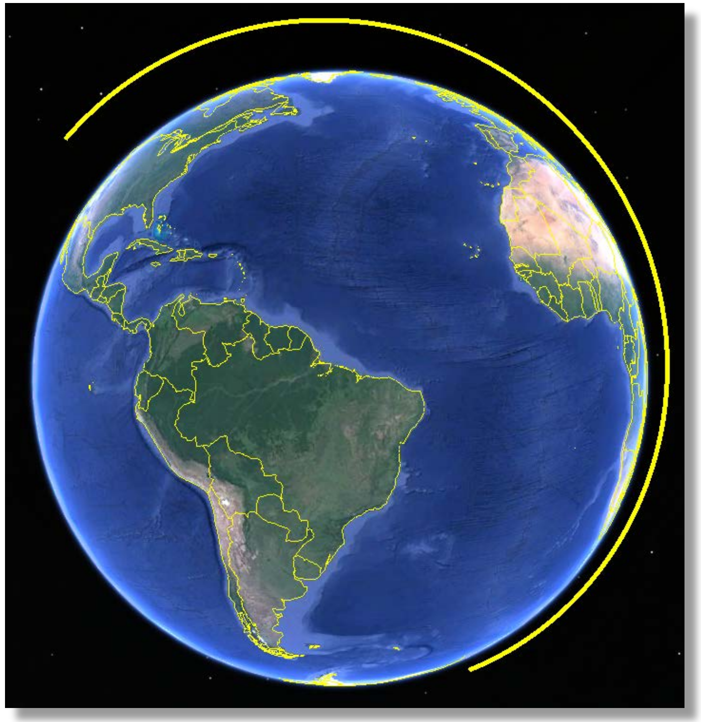
  $where \quad V_{avg} = Approximate\ instantaeous\ velocity$

  $R_r = Receiver\ position\ at\ epoch\ k$

  $\Delta t = Time\ interval$

  - Short time interval and low dynamic environment
  - Metre/decimetre-level positioning accuracy (SPP/PPP)
  - Velocity accuracy of <u>metres/decimetres-per-second</u>
    - Correspondingly-accurate accelerations

- UNB's precise point positioning (PPP) utility
  - http://gaps.gge.unb.ca/
  - Decimetre-level positioning accuracy
    - Kinematic-mode (epoch-by-epoch)
    - Dual-frequency ionosphere-free observables
      - GPS L_1/L_2 carrier-phase and pseudorange
  - Velocity determination via position change
    - Accuracies of decimetres-per-second



Velocity Estimation

o Receiver-Generated Doppler

- $D_{i_s} = \dot{P}_k^j = \frac{R^s - R_r}{\|R^s - R_r\|} \cdot (v^s - v_r) + \dot{B}_k + \varepsilon_k^j$

where $v^s = Satellite\ velocity\ vector$ $v_r = Receiver\ velocity\ vector$

$R^s = Satellite\ position\ vector$ $R_r = Receiver\ position\ vector$

$\dot{B}_k = Receiver\ clock\ drift$ $\varepsilon_k^j = Unmodeled\ errors$

- Pseudorange-rate ($\dot{P}$)
    - Receiver tracking loop noise
- Velocity accuracies of centimetres-per-second
    - Correspondingly-accurate accelerations

Velocity Estimation

- **Carrier-phase-derived Doppler (TDCP)** (Serrano, 2004)
  - First-order central-difference approximation
  - $D_{i_s} = \dot{\Phi}_k^j = \frac{\Phi_{k+\Delta t}^j - \Phi_{k-\Delta t}^j}{2\Delta t} \approx \frac{R^s - R_r}{\|R^s - R_r\|} \cdot (v^s - v_r) + \dot{B}_k + \varepsilon_k^j$

$$\begin{aligned} &where \quad \Phi_k^j = Carrier - phase\ observable \quad && \Delta t = Time\ interval \\ &v^s = Satellite\ velocity\ vector && v_r = Receiver\ velocity\ vector \\ &R^s = Satellite\ position\ vector && R_r = Receiver\ position\ vector \\ &\dot{B}_k = Receiver\ clock\ drift && \varepsilon_k^j = Unmodeled\ errors \end{aligned}$$

  - Velocity accuracy of <u>millimetres-per-second</u>
    - Correspondingly-accurate accelerations

Proposed Position/Velocity/Acceleration Algorithm

- Position (kinematic precise orbit determination)
  - Standard PPP algorithm
  - Dual-frequency ionosphere-free observables
    - GPS L_1/L_2 carrier-phase and pseudorange
      - $\Phi_{IF}^{k} = P + c(dT - dt) + \lambda_{IF}N_{IF} + \varepsilon_{IF}$
      - $P_{IF}^{k} = P + c(dT - dt) + \varepsilon_{IF}$
  - Decimetre-level position accuracies
    - Must be < 10 metres
- Velocity
  - Carrier-phase-derived Doppler (TDCP)

Proposed Position/Velocity/Acceleration Algorithm

- <u>Acceleration</u>
  - First-order central difference approximation
    - $\ddot{\Phi}^{j}_{k-\Delta t} = \frac{\dot{\Phi}^{j}_{k+\Delta t} - \dot{\Phi}^{j}_{k-\Delta t}}{2\Delta t} = \frac{\Phi^{j}_{k+2\Delta t} - 2\Phi^{j}_{k} + \Phi^{j}_{k-2\Delta t}}{4\Delta t^2}$

$$\approx \frac{v^s - v_r}{\|v^s - v_r\|} \cdot (v^s - v_r) + \frac{R^s - R_r}{\|R^s - R_r\|} \cdot (a^s - a_r) + \ddot{B}_k + \varepsilon\,^{j}_{k}$$

  - Dual-frequency ionosphere-free observables
    - GPS L_1/L_2 carrier-phase
  - Millimetre-per-second^2 accuracies

Thermospheric Density Variation

- Analysis of precise spacecraft acceleration estimates
  - Discrimination of forces acting on the satellite/GPS observables
    - Gravitational
    - Solar radiation pressure
    - **Thermospheric/Ionospheric particle density variation**
  - Methods similar to existing Swarm algorithms
    - P12: Thermosphere (poster): Doornbos, E., *GPS-derived Density Data for the Swarm Satellites During the Declining Phase of the Solar Cycle*
  - **Alternative approaches**
    - Krishnan, S. (2012). *Assessment of numerical differentiation methods for kinematic orbit solution of the GRACE mission.*

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For more CASSIOPE-based research from UNB, please see:

P05: Ionosphere: Nicholson, H. and R.B. Langley, *Determination of CASSIOPE Topside Ionospheric Total Electron Content Using GPS Precise Point Positioning Techniques* (poster)

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